$$\frac{\pi}{4}$$

1. Find $f \circ g$ and $g \circ f$ for the pair of functions

$$f(x) = \sqrt{x+2}, g(x) = 2x^2 - 1$$

Solution:

$$f \circ g = \sqrt{(2x^2+1)}, \quad g \circ f = 2(x+2) - 1 = 2x + 3$$

2. Solve. Leave answers in exact form

$$8 = 4^{x^2} * 2^{5x}$$

Solution:

$$2^3 = 2^{2x^2} * 2^{5x}$$
$$3 = 2x^2 + 5x$$
$$2x^2 + 5x - 3 = 0$$
$$(2x-1)(x+3) = 0$$
$$\therefore 2x - 1 = 0, x + 3 = 0$$
$$x = \frac{1}{2}, x = -3$$

3. Find $f \circ g$ and $g \circ f$ for the pair of functions

$$f(x) = 1 - 3x^2, g(x) = \sqrt{4-x}$$

Solution:

$$f \circ g = 1 - 3(4-x) = 3x - 11$$
$$g \circ f = \sqrt{4-(1-3x^2)} = \sqrt{3x^2+3}$$

4. Find the maximum value of

$$f(x) = -2x^2 + 4x - 21$$

Solution:

Maximum at vertex $\Rightarrow x = \frac{-b}{2a}$

$$x = \frac{-b}{2a} = \frac{-4}{2(-2)} = 1$$
$$f(1) = -2(1)^2 + 4(1) - 21 = -19$$

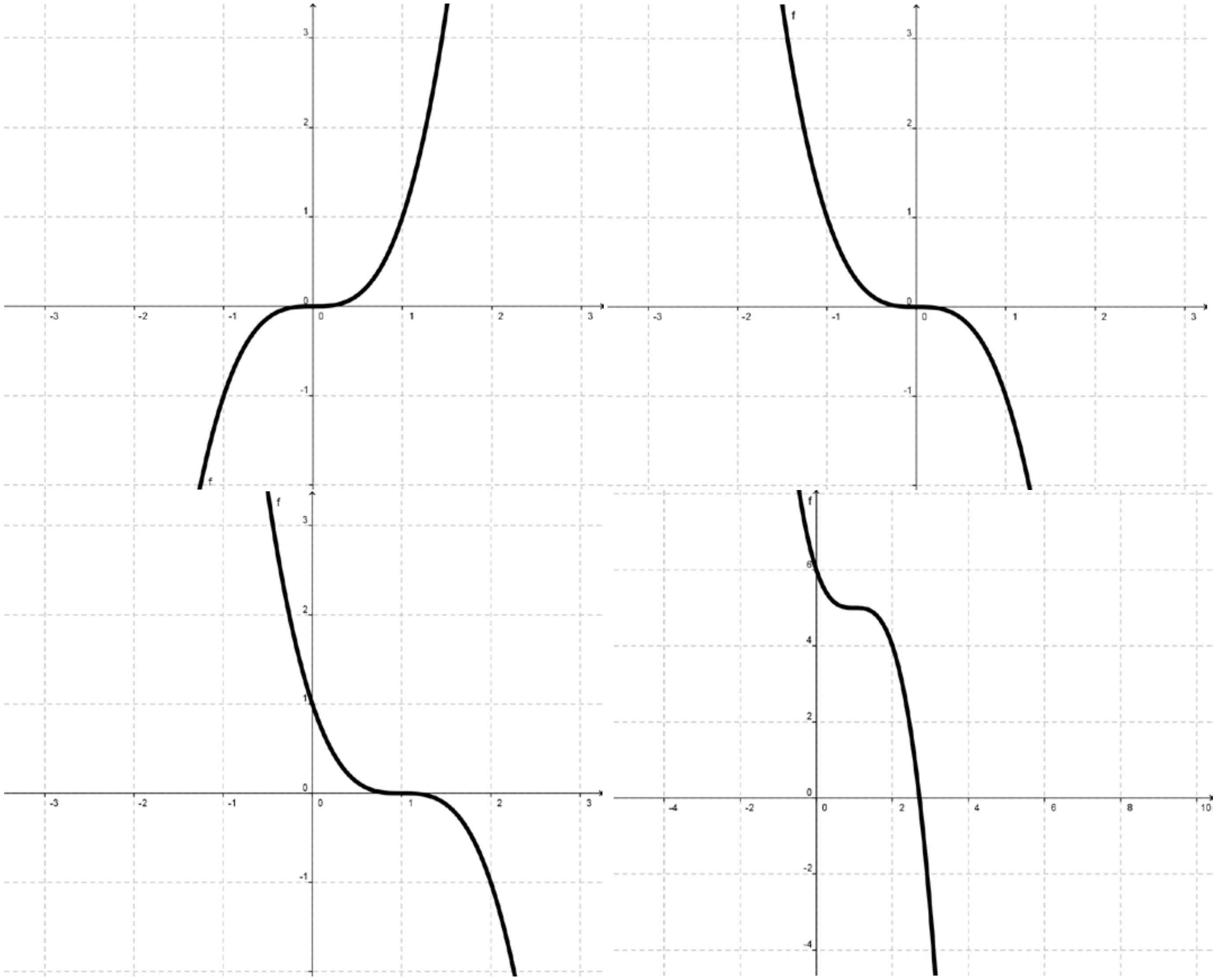
$\therefore$ max at $(1, -19)$

$$\boldsymbol{\frac{\pi}{2}}$$

1. Graph using transformations. Show all stages

$$f(x) = (1 - x)^3 + 5$$

Solution:



2. Write the expression as a single logarithm

$$2 \log 2 + 3 \log x - \frac{1}{2}(\log(x+3) + \log(x+2))$$

Solution: (Next page)

$$2\log 2 + 3\log x - \frac{1}{2}\left(\log\left(x+3\right) + \log\left(x+2\right)\right)$$
$$= \log 4 + \log x^3 - \frac{1}{2}\log\left((x+3)(x+2)\right)$$
$$= \log 4x^3 - \log\left((x+3)(x+2)^{\frac{1}{2}})\right)$$
$$= \log\left(\frac{4x^3}{(x+3)(x+2)^{\frac{1}{2}}}\right)$$

3. Determine the domain and find all the asymptotes

$$R(x) = \frac{(x^2 + 3x + 2)}{(x+2)^2}$$

Solution:

$D = \{x \mid x \in \mathbb{R}, x \neq -2\}$ or x all reals except -2

Asymptotes:

Vertical:

$x = -2$

Horizontal:

$$R(x) = \frac{\left(x^2 + 3x + 2\right)}{\left(x+2\right)^2} = \frac{(x+2)(x+1)}{(x+2)^2} = \frac{x+1}{x+2}$$

Since the $\left(\text{degree of the numerator}\right) = (\text{degree of the denominator})$,

we look for the coefficients of the highest degree term

$\text{HA} = y = \frac{1}{1} = 1$

4. Solve the inequality

$$x(x+1) > 20$$

Solution:

$x(x+1) > 20$

$x^2 + x - 20 > 0$

$(x+5)(x-4) > 0$

look at the following intervals and pick test values:

$(-\infty, -5), (-5, 4)(4, \infty)$

$(-\infty, -5) \to -6 \to 10 > 0$ YES

$(-5, 4) \to 0 \to -20 < 0$ NO

$(4, \infty) \to 5 \to 25 > 0$ YES

Hence we conclude this is true for x in the intevals $(-\infty, -5)$ and $(4, \infty)$

or we can also write $x \in (-\infty, -5) \cup (4, \infty)$

$$\frac{3\pi}{4}$$

1. Determine the domain and find all asymptotes

$$R(x) = \frac{x^2 + 4}{x - 2}$$

Solution:

$D = \{x \mid x \in \mathbb{R}, x \neq 2\}$ or x all reals except 2

Asymptotes:

Vertical:

$x = 2$

Horizontal:

$$R(x) = \frac{x^2 + 4}{x - 2}$$

Since the $(\text{degree of the numerator}) > (\text{degree of the denominator})$,

by one, we will have an oblique asymptote:

$$\begin{array}{r|l} & x + 2R8 \\ x - 2 & x^2 + 0x + 4 \\ & \underline{x^2 - 2x} \\ & 2x + 4 \\ & \underline{2x - 4} \\ & 8 \end{array}$$

$\therefore OA = y = x + 2$

2. Jered has 3500 feet of ice available to enclose a rectangular igloo. Express the area A of the rectangle as a function of x, where x is the length of the rectangle.

Solution:

$$P = 2x + 2y = 3500$$

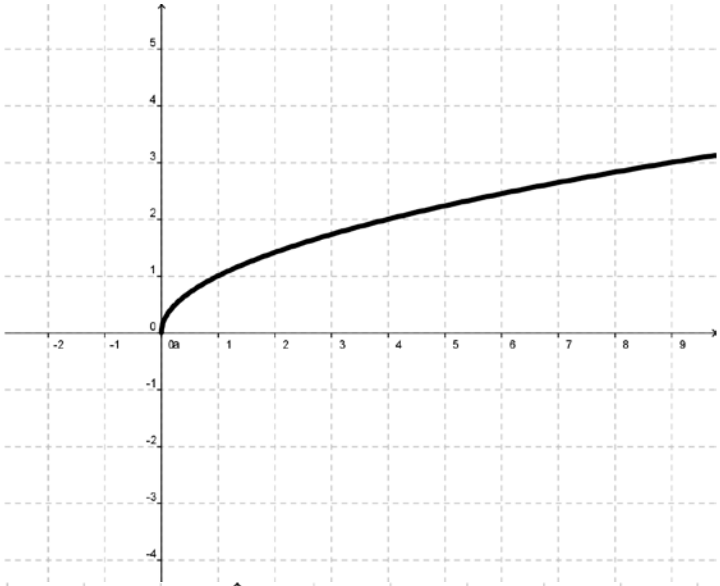
$$y = \frac{3500 - 2x}{2} = 1750 - x$$

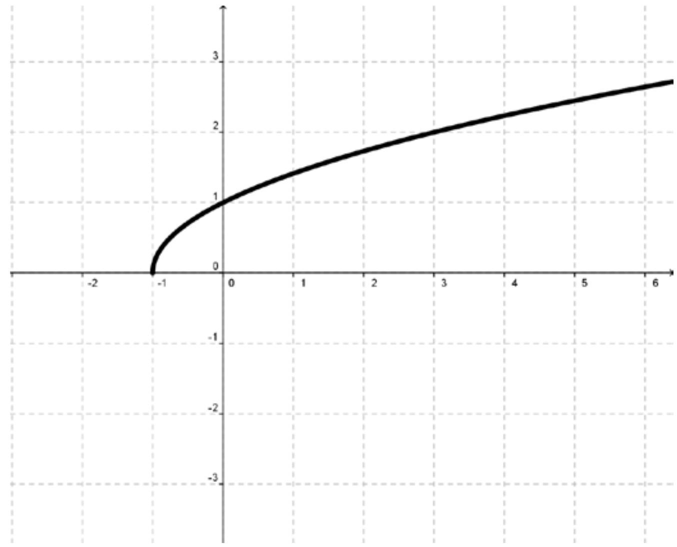
$$A = xy = x(1750 - x) = 1750x - x^2$$

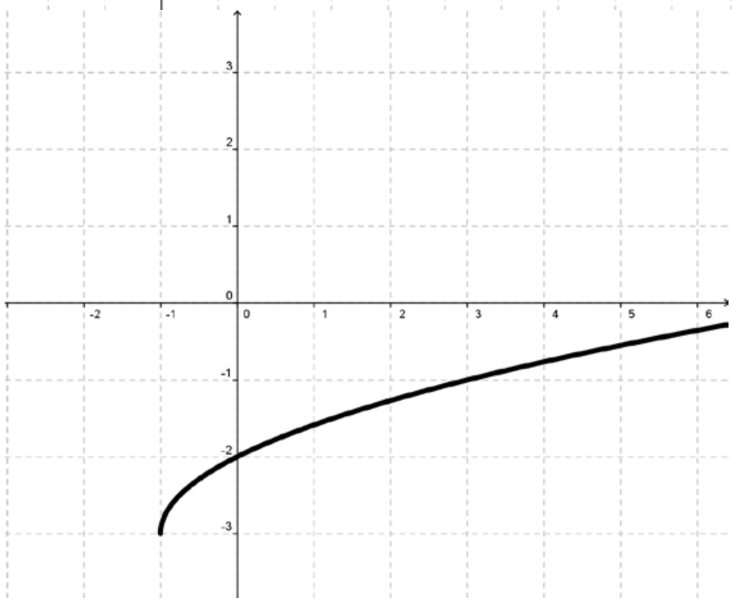
3. Graph using transformations. Show all stages

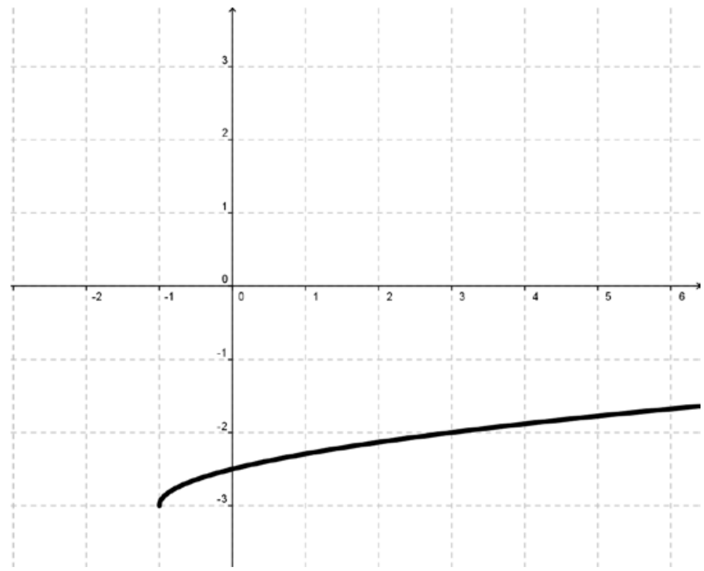
$$g(x) = \frac{1}{2}\sqrt{x + 1} - 3$$

Solution: (Next page)









4. Solve the inequality

$$x^2 + 7x < -12$$

Solution:

$$x^2 + 7x < -12$$

$$x^2 + 7x + 12 < 0$$

$$(x+3)(x+4) < 0$$

look at the following intervals and pick test values:

$$(-\infty, -4), (-4, -3)(-3, \infty)$$

$$(-\infty, -4) \to -5 \to 2 > 0 \text{ NO}$$

$$(-4, -3) \to -3.5 \to -\frac{1}{4} < 0 \text{ YES}$$

$$(-3, \infty) \to 0 \to 12 > 0 \text{ NO}$$

Hence we conclude this is true for x in the inteval $(-4, -3)$

or we can also write $x \in (-4, -3)$

$\boldsymbol{\pi}$

1. Solve each inequality. Graph the solution set.

$$\frac{x^2 - 8x + 12}{x^2 - 16} > 0$$

Solution:

$$\frac{x^2 - 8x + 12}{x^2 - 16} > 0$$
$$\frac{(x-6)(x-2)}{(x-4)(x+4)} > 0$$

look at the intervals and select test values

$$(-\infty, -4) \to -5 \to \frac{77}{9} > 0 \text{ YES}$$
$$(-4, 2) \to 0 \to \frac{12}{-16} < 0 \text{ NO}$$
$$(2, 4) \to 3 \to \frac{-3}{-7} = \frac{3}{7} > 0 \text{ YES}$$
$$(4, 6) \to 5 \to \frac{-3}{9} < 0 \text{ NO}$$
$$(6, \infty) \to 7 \to \frac{5}{33} > 0 \text{ YES}$$

Therefore,

$$x \in (-\infty, -4) \cup (2, 4) \cup (6, \infty)$$

(this says x is in each of these intervals...
using less than/greater than works too!)

2. Solve. Leave answers in exact form

$$log_{\,6}(x+3) + log_{\,6}(x+4) = 1$$

Solution:

$$\log_6(x+3) + \log_6(x+4) = 1$$
$$\log_6((x+3)(x+4)) = 1$$
$$(x+3)(x+4) = 6^1$$
$$x^2 + 7x + 12 = 6$$
$$x^2 + 7x + 6 = 0$$
$$x = -1 \text{ or } x = -6$$

(skipping the factoring)

3. Write the expression as a sum or difference of logarithms

$$\ln\left(\frac{2x+3}{x^2-3x+2}\right)^2$$

Solution:

$$\begin{aligned}
&\ln\left(\frac{2x+3}{x^2-3x+2}\right)^2\\
&=2\ln\left(\frac{2x+3}{x^2-3x+2}\right)\\
&=2\left(\ln\left(2x+3\right)-\ln\left(x^2-3x+2\right)\right)\\
&=2\ln\left(2x+3\right)-2\ln\left(x^2-3x+2\right)
\end{aligned}$$

4. Determine the domain and range of the function,

$$f(x) = -2x^2 - x + 4$$

Solution:

$D =$ all reals

For range, as it is a downward parabola, the maximum y value will be determined by the vertex:

$$f\left(\frac{-b}{2a}\right)=f\left(\frac{-(-1)}{2(-2)}\right)=f\left(\frac{-1}{4}\right)=\frac{33}{8}$$

$R =$ all y less than or equal to $\frac{33}{8}$ or $\left\{y \mid y\le\frac{33}{8}\right\}$

$$\mathbf{\frac{5\pi}{4}}$$

1. Find the ~~minimum~~ maximum value of

$$f(x) = -9x^2 - 6x + 3$$

Solution:

$$\text{Maximum at vertex } \Rightarrow x = \frac{-b}{2a}$$

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(-9)} = \frac{-6}{18} = \frac{-1}{3}$$

$$f\left(\frac{-1}{3}\right) = -9\left(\frac{-1}{3}\right)^2 - 6\left(\frac{-1}{3}\right) + 3 = 4$$

$$\therefore \text{ max at } \left(\frac{-1}{3}, 4\right)$$

2. Find the inverse of the function and ~~verify your answer~~

$$f(x) = \frac{x^2+3}{3x^2}$$

Solution:

$$f(x) = \frac{x^2+3}{3x^2}$$

$$f^{-1}(x) = x = \frac{y^2+3}{3y^2}$$

$$3xy^2 = y^2 + 3$$

$$y^2(3x-1) = 3$$

$$y^2 = \frac{3}{3x-1}$$

$$f^{-1}(x) = y = \pm\sqrt{\frac{3}{3x-1}}$$

3. Determine the domain and range of the function,

$$f(x) = \frac{x^2 - x - 12}{2x^2 + 8x + 6}$$

Solution:

$$f(x) = \frac{x^2 - x - 12}{2x^2 + 8x + 6} = \frac{(x-4)(x+3)}{(2x+2)(x+3)} = \frac{x-4}{2x+2}$$

$$D = \{x \mid x \in \mathbb{R}, x \neq -1, -3\}$$

$$R = \text{all reals}$$

4. Solve each inequality. Graph the solution set.

$$\frac{3-2x}{2x+5} \geq 2$$

Solution:

$$\frac{3-2x}{2x+5} \geq 2$$
$$3-2x \geq 4x+10$$
$$-7 \geq 6x$$
$$\frac{-7}{6} \geq x$$

$$\mathbf{\frac{3\pi}{2}}$$

1. Solve:

$$2(2x^2 - 3x) > +9$$

Solution:

$$2\left(2x^2 - 3x\right) > 9$$
$$4x^2 - 6x > 9$$
$$x^2 - \frac{3}{2}x > \frac{9}{4}$$
$$x^2 - \frac{3}{2}x + \frac{9}{16} > \frac{45}{16}$$
$$\left(x - \frac{3}{4}\right)^2 > \frac{45}{16}$$
$$x - \frac{3}{4} > \frac{3\sqrt{5}}{4} \text{ or } x - \frac{3}{4} < -\frac{3\sqrt{5}}{4}$$
$$x > \frac{3+3\sqrt{5}}{4} \text{ or } x < \frac{3-3\sqrt{5}}{4}$$

2. Solve each inequality. ~~Graph the solution set.~~

$$x^3 + x^2 < 4x + 4$$

Solution:

$$x^3 + x^2 < 4x + 4$$
$$x^2(x+1) < 4(x+1)$$
$$x^2(x+1) - 4(x+1) < 0$$
$$(x+1)(x^2 - 4) < 0$$
$$(x+1)(x+2)(x-2) < 0$$
$$\Rightarrow x < -1, x < -2, x < 2$$

After testing points in the intervals, you find the solution is:

$$x \in (-\infty, -2) \cup (-1, 2)$$

3. Find all intercepts, asymptotes, ~~end behavior~~, and behavior between zeros/asymptotes to obtain the graph of the rational function. (Seven step process)

$$R(x) = \frac{x+2}{x(x-2)}$$

Solution:

x intercepts

$0 = \dfrac{x+2}{x(x-2)} \Rightarrow x = -2$

y intercepts $\Rightarrow$ none

VA:

$x = 0, x = 2$

HA/OA:

As the degree of the denominator is greater than the degree of the numerator by one, we will have a horizontal asymptote at y=0. Does the function cross it?

Yes! at $x = -2$

Behavior between zeros/asymptotes

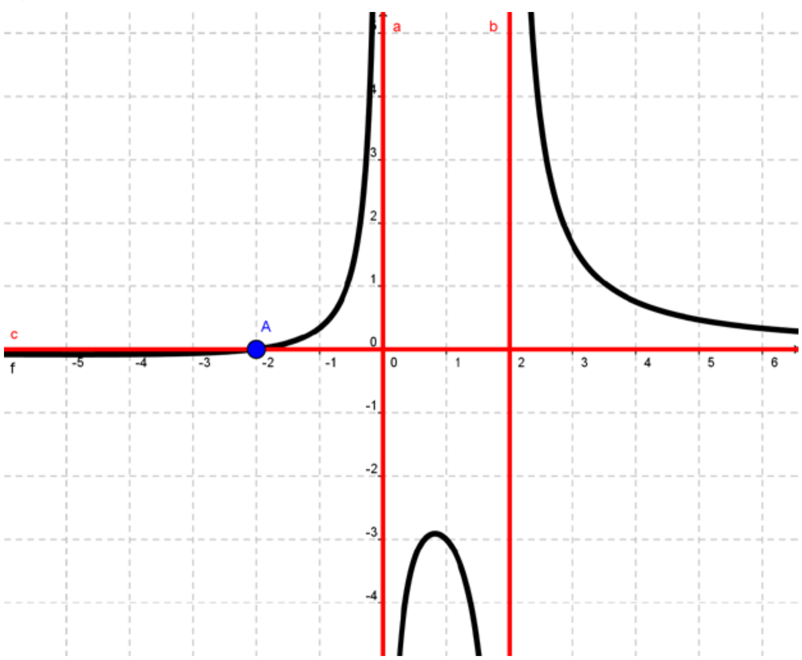
Look at:

$(-\infty, -2) \rightarrow -3 \rightarrow -$

$(-2, 0) \rightarrow -1 \rightarrow +$

$(0, 2) \rightarrow -1 \rightarrow -$

$(2, \infty) \rightarrow 3 \rightarrow +$



4. Determine the end behavior of

$$(\boldsymbol{a})\ f(x) = \frac{1}{x}, \qquad (\boldsymbol{b})\ f(x) = x^{n^n}, \quad (\boldsymbol{c})\ f(x) = \sqrt{x}$$

Solution:

(a) both approach zero as x goes to + and – infinity. (b) approaches infinity VERY fast for some real number n. (c) approaches infinity for x greater than 0.

$$\boldsymbol{\frac{7\pi}{4}}$$

1. Solve. Leave answers in exact form

$$3^{2x}+3^{x+1}-4=0$$

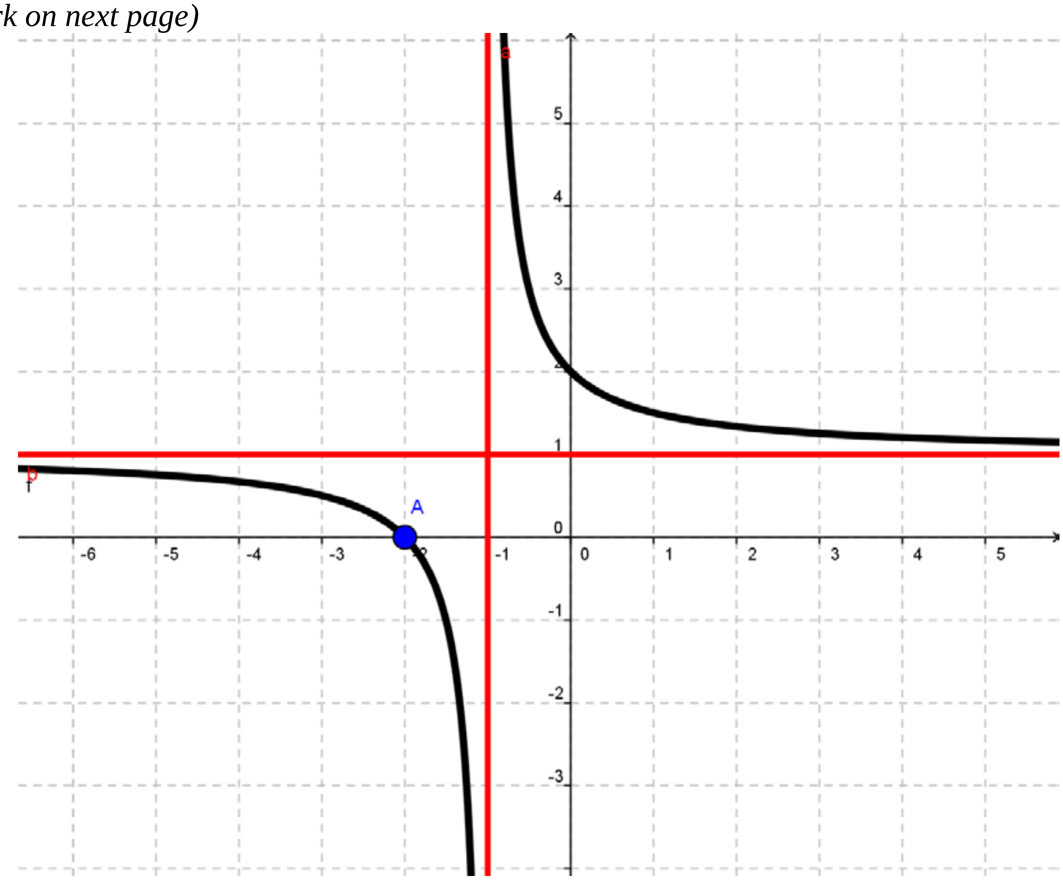
Solution:

$$3^{2x}+3^{x+1}-4=0$$
$$3^{2x}+3\bullet 3^{x}-4=0$$
$$\text{let } u=3^{x}$$
$$u^{2}+3u-4=0$$
$$(u+4)(u-1)=0$$
$$u=-4, u=1$$
$$3^{x}=-4, 3^{x}=1$$
$$x\log 3=\log(-4), x\log 3=\log 1$$
$$x=\frac{\log(-4)}{\log 3}, x=\frac{0}{\log 3}$$
$$DNE, x=0$$
$$\therefore x=0$$

2. Find all intercepts, asymptotes, ~~end behavior~~, and behavior between zeros/asymptotes to obtain the graph of the rational function. (Seven step process)

$$H(x)=\frac{(x^{2}-4)}{x^{2}-x-2}$$

Solution: (work on next page)



$$H(x)=\frac{x^2-4}{x^2-x-2}=\frac{(x-2)(x+2)}{(x-2)(x+1)}=\frac{x+2}{x+1}$$

HOLE AT $x=2$

x intercepts

$$0=\frac{x^2-4}{x^2-x-2}\Rightarrow x=-2$$

y intercepts

$$y=\frac{-4}{-2}=2$$

VA:

$x=-1$

HA/OA:

As the degree of the denominator is equal to the degree of the numerator, we will have a horizontal asymptote at y=$\frac{1}{1}$=1. Does the function cross it?

$$\frac{x+2}{x-1}=1$$

$x-1=x-2$

$-1=-2$

As this is false, it will NOT cross the asymptote.

Behavior between zeros/asymptotes

Look at:

$(-\infty,-2)\to-3\to+$

$(-2,-1)\to-1.5\to-$

$(-1,2)\to0.5\to+$

$(2,\infty)\to3\to+$

3. Find the inverse of the function ~~and verify your answer~~

$$f(x)=\frac{2-x}{3+x}$$

Solution:

$$f(x)=y=\frac{2-x}{3+x}$$

$$f^{-1}(x)=x=\frac{2-y}{3+y}$$

$$3x+xy=2-y$$

$$y(x+1)=2-3x$$

$$y=\frac{2-3x}{x+1}=f^{-1}(x)$$

4. On Thursday, Vishal decides to slingshot your final exams to each one of you upon your entrance to the final exam room. His old-school slingshot fires the exams at an inclination of 45° to the horizontal with a velocity of 25 ft. sec. The height is given by

$$h(x) = \frac{-32x^2}{(25)^2} + x$$

where x is the horizontal distance of the exam from Vishal. Find the maximum height the exam will reach.

Solution:

$$\text{Maximum at vertex } \Rightarrow x = \frac{-b}{2a}, y = f\left(\frac{-b}{2a}\right)$$

$$x = \frac{-b}{2a} = \frac{-(1)}{2\left(\frac{-32}{25^2}\right)} = \frac{25^2}{64} = \frac{625}{64}$$

$$f\left(\frac{625}{64}\right) = \frac{-32}{25^2}\left(\frac{625}{64}\right)^2 + \left(\frac{625}{64}\right) = \frac{625}{128}$$

$$\therefore \text{max height} = \frac{625}{128}\, ft$$

2π

1. Find all intercepts, asymptotes, ~~end behavior~~, and behavior between zeros/asymptotes to obtain the graph of the rational function. (Seven step process)

$$G(x) = \frac{x^4}{x^2 - 9}$$

Solution:

$$H(x) = \frac{x^4}{x^2 - 9} = \frac{x^4}{(x-3)(x+3)}$$

x intercepts

$$0 = \frac{x^4}{(x-3)(x+3)} \Rightarrow x = 0$$

y intercepts

y=0

VA:

$x = -1$

HA/OA:

As the degree of the numerator is more than two degrees of the denominator, we will not have a HA/OA.

Behavior between zeros/asymptotes

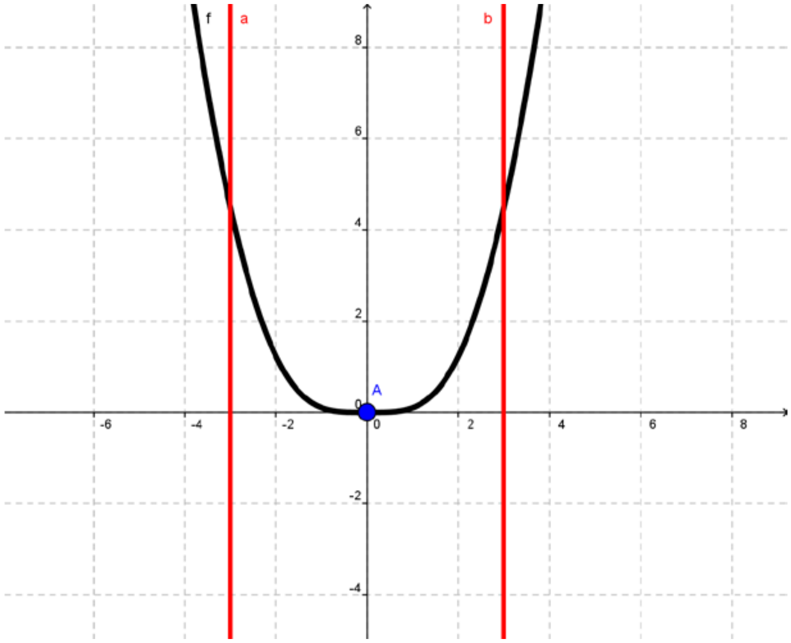
Look at:

$(-\infty, -3) \rightarrow -4 \rightarrow +$

$(-3, 0) \rightarrow -1 \rightarrow +$

$(0, 3) \rightarrow 1 \rightarrow +$

$(3, \infty) \rightarrow 4 \rightarrow +$



****Note: there are holes at $x = 3, -3$**

2. A child's grandparents purchase a $10,000 bond fund that matures in 18 years to be used for her college education. The bond fund pays 4% interest compounded semiannually. How much will the bond be worth at maturity? How long will it take the bond to double in value under these terms?

Solution:

$$P = 10,000$$
$$t = 18$$
$$r = 0.04$$
$$n = 2$$
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$A = 10000(1.02)^{36} = 20,398.87 \text{ dollars}$$

3. A friend and you decide to start up a used textbook business. You must pay in $400 dollars to start the business. You then buy used PreCalculus books from math students for $40 each. You turn around and sell them to incoming freshman the following semester for $60 each. How many books must you sell to "break even"? (In other words, how many books must you sell before your business is out of the debt from paying the 400 dollar buy in and $40 per book?)

Solution:

Let x=number of books

If you want to break even, profit=expenses

$$60x = 400 + 40x$$
$$20x = 400$$
$$x = 20 \text{ books}$$

4. Determine whether the graph opens up or down, and find its vertex, axis of symmetry, y-intercept, and x-intercepts, if any

$$f(x) = 3x^2 + 12x + 1$$

Solution:
Opens up as quadratic coefficient is positive

$$\text{vertex } \Rightarrow x = \frac{-b}{2a}, y = f\left(\frac{-b}{2a}\right)$$

$$x = \frac{-b}{2a} = \frac{-(12)}{2(3)} = -2$$

$$f(-2) = -11$$

$$\text{vertex } \Rightarrow (-2, -11)$$

$$\text{axis of symmetry: } x = \frac{-b}{2a} = -2$$

x intercept:

$$0 = 3x^2 + 12x + 1$$

$$-1 = 3(x^2 + 4x)$$

$$11 = 3(x^2 + 4x + 4)$$

$$\frac{11}{3} = (x+2)^2$$

$$x = -2 \pm \sqrt{\frac{11}{3}}$$

y intercept:

$$y = 1$$